Response of Laser Heat Exchangers to Unsteady Spatially Varying Input

A. K. Cousins*
Spectra Technology Inc., Bellevue, Washington 98004

The response of pulsed gas laser heat exchangers to periodic temperature input is analyzed theoretically, and an analytical solution for the decay of thermal pulses with heat exchanger length is presented. A closed-form solution for the response of thermalizers to unsteady and spatially varying input is also developed.

Nomenclature

C_c = cold fluid capacity rate, W/K
C_c' = cold fluid fixed capacity, J/K
C_h = hot fluid capacity rate, W/K
C_h' = hot fluid fixed capacity, J/K
C_{max} = maximum of (C_c, C_h)
C_{min} = minimum of (C_c, C_h)
C_w' = wall fixed capacity, J/K
f_p = laser pulse repetition frequency
K = number of thermalizer flow passages
L = heat exchanger length in flow direction
R_c = total heat transfer resistance (K/W) between wall and cold fluid
R_h = total heat transfer resistance (K/W) between wall and hot fluid
St = Stanton number
T_c = cold fluid (coolant) temperature
T_h = hot fluid (gas) temperature
T_w = wall temperature
α_c = LC_c/C_c'
α_h = LC_h/C_c'
β_c = $1/(C_c'R_c)$
β_h = $1/(C_h'R_h)$
β_{wc} = $1/(C_w'R_c)$
β_{wh} = $1/(C_w'R_h)$
$\delta_c(\omega)$ = see Eq. (10)
$\delta_h(\omega)$ = see Eq. (10)
Δ = diameter (fin spacing) of an individual thermalizer flow passage
$\gamma_c(\omega)$ = see Eq. (9)
$\gamma_h(\omega)$ = see Eq. (9)
ω_n = $2\pi n f_p$

Introduction

RESTORATION of laser gas temperature uniformity is a critical requirement in closed-loop, pulsed discharge lasers and is brought about by the incorporation of a gas-to-liquid heat exchanger and possibly by a thermalizer into the laser gas flow loop. Cassady[1] gives a review of design issues in compact, laser-flow loops to which the reader is referred for background information. As a result of the periodic discharges in the laser cavity, a train of alternating hot and cold gas regions is convected around the flow loop to the heat exchanger (see Fig. 1). In general, these temperature variations are nonuniform in both space and time and must typically be attenuated from initial levels of the order of 100 to 0.1 K or less before being convected back into the discharge region.

The present work addresses these questions of spatial and temporal nonuniformity in inlet conditions. The motivation for the study is that an understanding of the heat transfer characteristics of laser heat exchangers can lead to significant improvements in their compactness, weight, and coolant flow requirements.

Although there is a large body of work in the literature on the transient response of heat exchangers,[2-7] most of the work of which the author is aware deals with initial transients and not the steady periodic conditions which occur in a laser-flow loop (with the exception of work on periodic-flow regenerators, which are a somewhat different case). Paynter and Takahashi,[7] for example, present rather general transient solutions in Laplace-transformed form; the solutions are so general, however, that they can be inverted to the time domain in approximate form only. Other authors[2] have presented step-function response curves and correctly state that all other transient inlet conditions can be synthesized from these data using Duhanel's theorem; however, this is an extremely difficult way to approach a steady periodic problem—especially since most of the step-function response data are given in numerical or graphical form. Finally, to the best of the author's knowledge, the bulk of the existing work does not consider the effects of heat transfer between adjacent gas flow passages in a heat exchanger or thermalizer subjected to spatially nonuniform input temperature.

At the time of publication, the author was apprised by a referee of a closely related work by Kulkarny et al.[8] The

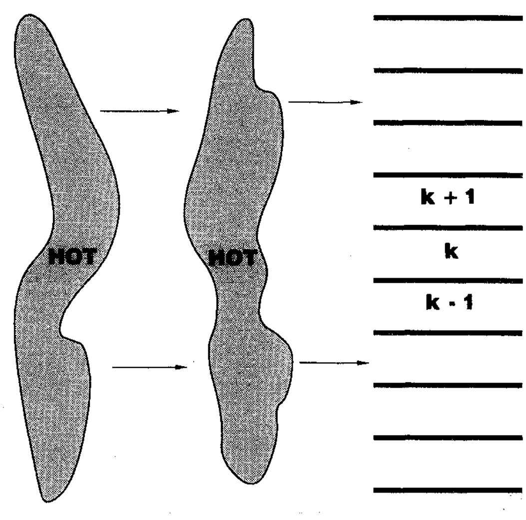

Fig. 1 Alternating hot and cold gas regions arriving at heat exchanger inlet.

Received May 9, 1989; revision received Sept. 8, 1989; accepted for publication Oct. 4, 1989.

*Senior Scientist, Flow Technology Group. Member AIAA.

approach of Ref. 8 differs from the present work in that the basic equations are averaged over a number of flow passages, leading to an approximate partial differential equation rather than the differential-difference equation solved here [Eq. (26)]. However, the primary conclusions of Ref. 8 are in agreement with those presented here.

Mathematical Model

The major assumptions used in developing the model are identical to those used in standard steady-state heat exchanger theory[3] and are listed as follows.

1) The fluid and wall temperatures are functions of time and one spatial dimension (the flow direction). In the thermalizer analysis, temperatures are also allowed to vary in the direction transverse to the flow and the thermalizer fins.

2) The heat transfer between the fluids and the solid walls is modeled by lumped thermal resistances (heat transfer coefficients).

3) The mass flow rates of both fluids are assumed to be constant.

4) Conduction of heat along the walls in the flow direction is neglected relative to fluid-to-wall heat transfer. This assumption is only necessary in the case of steady-state heat transfer ($\omega = 0$). In the following discussion, it will be demonstrated that in the periodic case, the wall temperature is not changed appreciably by the gas temperature oscillations. Thus longitudinal conduction in the walls is not relevant to the calculation of the gas temperature.

In addition to these standard assumptions, it will be assumed that the cross-parallel flow heat exchanger can be represented by a purely parallel flow model. Kays and London[2] show that this approximation is valid for small values of the gas-to-liquid capacity ratio; this requirement is satisfied in laser applications (see the example calculation below). In fact, in the following discussion, it will be shown that the coolant flow configuration has no bearing on the unsteady heat transfer characteristics of a laser heat exchanger (although it certainly is important in *steady* heat transfer).

Given the preceding assumptions, the relevant conservation equations can be developed from an energy balance on a differential element of the heat exchanger. These equations are presented below following Ref. 3:

$$\frac{\partial T_h}{\partial t} + \alpha_h \frac{\partial T_h}{\partial x} = \beta_h (T_w - T_h) \tag{1}$$

$$\frac{\partial T_c}{\partial t} + \alpha_c \frac{\partial T_c}{\partial x} = \beta_c (T_w - T_c) \tag{2}$$

$$\frac{\partial T_w}{\partial t} = \beta_{wh}(T_h - T_w) + \beta_{wc}(T_c - T_w) \tag{3}$$

At the inlet to the heat exchanger ($x = 0$), the gas temperature is typically a train of square wave pulses in time, which can be represented by a Fourier expansion:

$$T(0,t) = \sum_{n=-\infty}^{\infty} A_n \exp(i\omega_n t) \tag{4}$$

This initial condition suggests a solution for the temperature response with the same time dependence:

$$T(x,t) = \sum_{n=-\infty}^{\infty} T_n(x) \exp(i\omega_n t) \tag{5}$$

Solutions of this form are assumed for the wall temperature and both fluid temperatures. In the following development, the subscript n and the exponential time dependence are suppressed for notational simplicity.

Substituting the preceding assumed form into Eqs. (1–3) gives ordinary differential equations for the fluid temperatures and an algebraic equation (whose solution is given below) for the wall temperature:

$$\frac{dT_h}{dx} + \gamma_h(\omega) T_h = \delta_h(\omega) T_c \tag{6}$$

$$\frac{dT_c}{dx} + \gamma_c(\omega) T_c = \delta_c(\omega) T_h \tag{7}$$

$$T_w = \frac{\beta_{wh} T_h + \beta_{wc} T_c}{i\omega + (\beta_{wh} + \beta_{wc})} \tag{8}$$

Note that Eq. (8) has been used to simplify Eqs. (6) and (7). The constants $\delta_h(\omega)$ and $\gamma_h(\omega)$ in Eq. (6) are defined by

$$\gamma_h(\omega) \equiv \frac{\beta_h}{\alpha_h}\left[1 - \frac{\beta_{wh}(\beta_{wc} + \beta_{wh})}{\omega^2 + (\beta_{wc} + \beta_{wh})^2}\right] + \frac{i\omega}{\alpha_h}\left[1 + \frac{\beta_h \beta_{wh}}{\omega^2 + (\beta_{wc} + \beta_{wh})^2}\right] \tag{9}$$

$$\delta_h(\omega) \equiv \frac{\beta_h \beta_{wc}}{\alpha_h}\left[\frac{\beta_{wc} + \beta_{wh}}{\omega^2 + (\beta_{wc} + \beta_{wh})^2} - \frac{i\omega}{\omega^2 + (\beta_{wc} + \beta_{wh})^2}\right] \tag{10}$$

The analogous relations for $\gamma_c(\omega)$ and $\delta_c(\omega)$ are obtained by transposing the c and h subscripts in Eq. (9) and (10).

Equations (6) and (7) can be solved via Laplace transformations in x, defined as follows:

$$T^*(s) \equiv \int_0^\infty T(x)\, e^{-sx} dx \tag{11}$$

The transformed versions of Eqs. (6) and (7) are

$$[s + \gamma_h(\omega)] T_h^* = T_h(0) + \delta_h(\omega) T_c^* \tag{12}$$

$$[s + \gamma_c(\omega)] T_c^* = T_c(0) + \delta_c(\omega) T_h^* \tag{13}$$

Solving Eqs. (12) and (13) for the transform of the hot fluid temperature yields

$$T_h^*(s) = \frac{[s + \gamma_c(\omega)] T_h(0) + \delta_h(\omega) T_c(0)}{s^2 + [\gamma_h(\omega) + \gamma_c(\omega)]s + [\gamma_c(\omega) + \gamma_h(\omega) - \delta_c(\omega)\delta_h(\omega)]} \tag{14}$$

Again, an analogous equation holds for the transform of the cold fluid temperature.

The cases $\omega = 0$ and $\omega \neq 0$ will now be considered separately. For $\omega = 0$, i.e., steady-input fluid temperatures, we have

$$\gamma_h(0) = \delta_h(0) = \frac{\beta_h \beta_{wc}/\alpha_h}{\beta_{wc} + \beta_{wh}} \tag{15a}$$

$$\gamma_c(0) = \delta_c(0) = \frac{\beta_c \beta_{wh}/\alpha_c}{\beta_{wc} + \beta_{wh}} \tag{15b}$$

and Eq. (14) can be easily inverted to give the steady-state response to the mean hot gas temperature:

$$T_{0h}(x) = \frac{\gamma_h(0) T_{0c}(0) + \gamma_c(0) T_{0h}(0)}{\gamma_h(0) + \gamma_c(0)} + \frac{\gamma_h(0)[T_{0h}(0) - T_{0c}(0)]}{\gamma_h(0) + \gamma_c(0)} \exp\{-[\gamma_c(0) + \gamma_h(0)]x\} \tag{16}$$

The restored subscript 0 indicated the zeroth harmonic, i.e., the mean component of the fluid temperatures. The first term in Eq. (16) represents the asymptotic temperature reached by both the hot and cold fluids in the limit as the heat exchanger becomes infinitely long, whereas the second term represents the decaying temperature difference between the hot fluid and this asymptotic temperature.

The decay of the nonzero wave number modes is the focus of this study and can be calculated exactly by inverting Eq. (14). However, a great deal of algebraic complexity can be obviated by examining the orders of magnitude of various terms. In typical laser applications

$$\beta_{wh}\beta_{wc} \ll \omega_1 \tag{17}$$

due to the large thermal capacity of the exchanger walls (see the next section for typical magnitudes). Equations (9) and (10) can then be simplified, for small (β_{wc}/ω) and (β_{wh}/ω), to

$$\gamma_h(\omega) \simeq \frac{\beta_h}{\alpha_h} + i\frac{\omega}{\alpha_h} \tag{18}$$

$$\delta_h(\omega) \simeq \frac{\beta_h}{\alpha_h}\left[\frac{\beta_{wc}(\beta_{wc}+\beta_{wh})}{\omega^2} - \frac{i\beta_{wc}}{\omega}\right] \tag{19}$$

Using this result, it can be shown that

$$\mathrm{Re}[\delta_c(\omega)\delta_h(\omega)] \ll \mathrm{Re}[\gamma_c(\omega)\gamma_h(\omega)] \tag{20a}$$

$$\mathrm{Im}[\delta_c(\omega)\delta_h(\omega)] \ll \mathrm{Im}[\gamma_c(\omega)\gamma_h(\omega)] \tag{20b}$$

The inverse of Eq. (14) is then simply

$$T_{nh}(x) = \exp\left[-\left(\frac{\beta_h}{\alpha_h} + i\frac{\omega_n}{\alpha_h}\right)x\right]T_{nh}(0), \qquad \text{all } n \neq 0 \tag{21}$$

Note that $T_{nc} = 0$ for $n \neq 0$ since the coolant inlet temperature is assumed to be steady in time.

The meaning of the preceding approximation is that the temporal period of oscillation of the unsteady temperature input (i.e., the laser pulse repetition period) is much shorter than the time required for heat transfer from either fluid to change the temperature of the exchanger walls. The implication of this is evident in Eq. (21): the unsteady component of the temperature input travels through the exchanger with unchanged shape (i.e., minimal dispersion) but exponentially decaying amplitude.

As far as the gas temperature oscillations are concerned, the heat exchanger walls represent an infinite thermal capacitance. In fact, this situation corresponds exactly to that in a thermalizer. Thermalizers are essentially heat exchangers that do not transfer heat to a coolant; they serve only to smooth temperature oscillations in the gas stream leaving a conventional heat exchanger and do not provide any net energy removal. Thus, Eq. (21) applies also to thermalizers used in laser applications.

Example Calculation

In order to give a sense of the numerical magnitudes involved, an actual laser design will be analyzed. The laser is a 150-W CO_2 laser operating at 50 HZ, and a 12-fin/in. aluminum heat exchanger is a cross-parallel flow configuration is used. The coolant in a 30% mixture of ethylene glycol in water. The relevant parameters for this laser are

Gas maximum inlet temperature = 401 K
Gas minimum inlet temperature = 300 K
Gas mean outlet temperature = 300 K
Coolant inlet temperature = 298 K
$1/R_h = 498$ W/K
$1/R_c = 1125$ W/K
$C_h = 98$ W/K
$C_c = 1306$ W/K
$C_w' = 8500$ J/K
$\omega_1 = 2\pi(50 \text{ Hz}) = 314 \text{ s}^{-1}$
Exchanger length = 8 cm

Based on these parameters, we can calculate
$\beta_{wh} = 5.86 \times 10^{-2}\text{s}^{-1}$
$\beta_{wc} = 1.32 \times 10^{-1}\text{s}^{-1}$
$\beta_h/\alpha_h = 6.33 \times 10^{-1}\text{cm}^{-1}$
Coolant outlet temperature = 299.5 K
Peak-to-peak gas temperature oscillations at outlet = 0.64 K

Thermalizer Response to Unsteady and Spatially Varying Input

The response of thermalizers will now be examined in a similar way; the analysis is slightly more general in that the thermalizer input temperature profile, as well as being unsteady, is allowed to have an arbitrary spatial variation transverse to the flow direction. These conditions are typical for actual thermalizers. In general, the thermalizer is downstream of the heat exchanger, whose thermal residue will exhibit spatial variations as well as temporal ones. There may be a mean gradient of temperature across the channel as a result to the heated gas having flowed around bends in the flow loop; in addition, there will be periodic thermal wakes from the heat exchanger fins due to the exit temperature difference between the fins and gas. Thermalizers act to smooth all of these variations.

The thermalizer is typically a series of parallel plates with gas flowing between them; in fact, the usual practice is to use a tube-and-plate crossflow heat exchanger without a coolant as a thermalizer. Equations similar to Eqs. (1-3) can be written; however, in this case they are written for an individual gas flow passage in the thermalizer in order to allow for transverse variation of the gas temperature. Figure 1 illustrates this geometry; assuming a total of K such parallel passages, the equations for the kth passage take the form of two coupled partial differential-difference equations:

$$\frac{\partial T_h^{(k)}}{\partial t} + \alpha_h\frac{\partial T_h^{(k)}}{\partial x} = \tfrac{1}{2}\beta_h(T_w^{(k)} - T_h^{(k)}) + \tfrac{1}{2}\beta_h[T_w^{(k+1)} - T_h^{(k)}] \tag{22}$$

$$\frac{\mathrm{d}T_w^{(k)}}{\mathrm{d}t} = \tfrac{1}{2}\beta_{wh}[T_h^{(k)} - T_w^{(k)}] + \tfrac{1}{2}\beta_{wh}[T_h^{(k-1)} - T_w^{(k)}] \tag{23}$$

If we again assume periodic temperature variations, the following equation for the gas temperature can be derived:

$$\alpha_h\frac{\mathrm{d}T_h^{(k)}}{\mathrm{d}x} + (i\omega + \beta_h)T_h^{(k)} = \frac{\beta_h\beta_{wh}/4}{i\omega + \beta_{wh}}[T_h^{(k+1)} + 2T_h^{(k)} + T_h^{(k-1)}] \tag{24}$$

This differential-difference equation can be solved in closed form. Two separate cases will be distinguished. The first case is $\omega \neq 0$, i.e., unsteady spatially varying gas input temperature. The approximation used in the heat exchanger analysis is invoked again, i.e., $\beta_{wh} \ll \omega$; the right side of Eq. (24) is therefore negligible, and the equation for the kth passage is uncoupled from the equations for the other passages. The solution to Eq. (24) is then

$$T_{nh}^{(k)} = T_{nh}^{(k)}(0)\exp\left[-\left(i\frac{\omega_n}{\alpha_h} + \frac{\beta_h}{\alpha_h}\right)x\right] \qquad (n \neq 0) \tag{25}$$

This result is analogous to Eq. (21) and implies that the nonsteady components of the input temperature distribution decay exponentially with distance along the thermalizer; the decay envelope is independent of the initial spatial distribution of temperature.

The second case arising from Eq. (24) is $\omega = 0$, i.e., the steady part of the nonuniform temperature distribution. In this case Eq. (24) reduced to

$$\frac{\mathrm{d}T_h^{(k)}}{\mathrm{d}x} = \frac{\beta_h}{4\alpha_h}[T_h^{(k+1)} - 2T_h^{(k)} + T_h^{(k-1)}] \qquad \omega = 0 \tag{26}$$

Note that this equation resembles a diffusion equation in two spatial dimensions whose right side has been discretized via central differencing. This leads us to expect that the initial transverse temperature distribution should diffuse away with increasing x.

The solution to Eq. (26) can be obtained by performing a discrete Fourier cosine transform[9] in the k direction. This transform and its inverse are defined by

$$\hat{f}(j) \equiv \frac{2}{K}\sum_{k=0}^{K-1} f(k)\cos jy_k \qquad \text{where } y_k \equiv \frac{(2k+1)\pi}{2K} \tag{27a}$$

$$f(k) \equiv \tfrac{1}{2}\hat{f}(0) + \sum_{j=1}^{K-1}\hat{f}(j)\cos jy_k \tag{27b}$$

Here K is the total number of gas flow passages in the thermalizer. The particular form of the cosine transform is chosen to satisfy the boundary conditions at the heat exchanger walls, which are assumed to be insulated. This implies that

$$T_h^{(-1)} = T_h^{(0)} \tag{28a}$$

$$T_h^{(K-1)} = T_h^{(K)} \tag{28b}$$

The preceding cosine transform satisfies these boundary conditions, as can be verified by direct substitution. The discrete Fourier transform of Eq. (19) is then, after simplification,

$$\frac{\mathrm{d}\hat{T}_h^{(j)}}{\mathrm{d}x} = \frac{\beta h}{2\alpha_h}\left[1-\cos\left(\frac{j\pi}{K}\right)\right]\hat{T}_h^{(j)}, \qquad j \neq 0$$

$$= 0, \qquad j = 0 \tag{29}$$

Solving this equation and performing the inverse discrete transform yields the final solution for the spatially varying temperature field:

$$T_h^{(k)}(x) = \frac{1}{K}\hat{T}_h^{(0)}(0) + \frac{2}{K}\sum_{j=1}^{K-1}\hat{T}_h^{(j)}(0)$$

$$\times \cos(y_k j)\exp\left\{-\frac{\beta_h}{2\alpha_h}\left[-\cos\left(\frac{j\pi}{K}\right)\right]x\right\} \tag{30}$$

Discussion

The response of heat exchangers to steady spatially uniform temperature inputs has been well characterized and can be calculated by standard methods.[2] The following discussion will focus on unsteady and spatially inhomogenous inlet conditions. Therefore, the solutions just derived for thermalizer will be examined; heat exchangers respond in essentially the same manner as thermalizers under these conditions.

Equation (25) represents the thermalizer response to the unsteady components of the inlet temperature distribution. These unsteady components undergo an exponential decay in amplitude as they travel through the thermalizer. It is useful to more that the decay constant (β_h/α_h) is related to the Stanton number by

$$\frac{\beta_h}{\alpha_h} = \frac{4St}{D_h} \tag{31}$$

where D_h is the hydraulic diameter of an individual passage (twice and fin spacing Δ in the flat-plate case) in the thermalizer, and St is the Stanton number. This form is convenient because heat exchanger heat transfer correlations generally give the Stanton number as as function of the gas Prandtl and Reynolds numbers and the exchanger core geometry.[2]

The thermalizer response to steady, spatially varying inlet temperature fields is given by Eq. (30). An exponential decay with distance is again evident, but the decay constant is dependent on the relative wave number (j/K) of the transverse temperature field. Evidently that higher wave number transverse modes decay more rapidly than the low wave number modes.

Figure 2 illustrates the predicted responses to both steady and unsteady inlet conditions. The curves are the decay envelopes predicted by Eqs. (25) and (30). Evidently the steady transverse modes always decay more slowly than the unsteady modes; however, high wave number steady transverse modes are attenuated almost as well as the unsteady modes in a thermalizer. As the value of the relative wave number (j/K) approaches one, the decay rate of the steady modes approaches that of the unsteady modes.

Figure 3 shows the decay of a steady linear transverse temperature gradient with downstream distance as a function of the relative transverse coordinate (k/K). Clearly, the temperature gradient decays extremely slowly with distance. In typical laser, the value of $x\beta_h/\alpha_h$ is much less than to (see the preceding example calculation and, therefore, steady transverse gradients decay minimally through the thermalizer. The reason

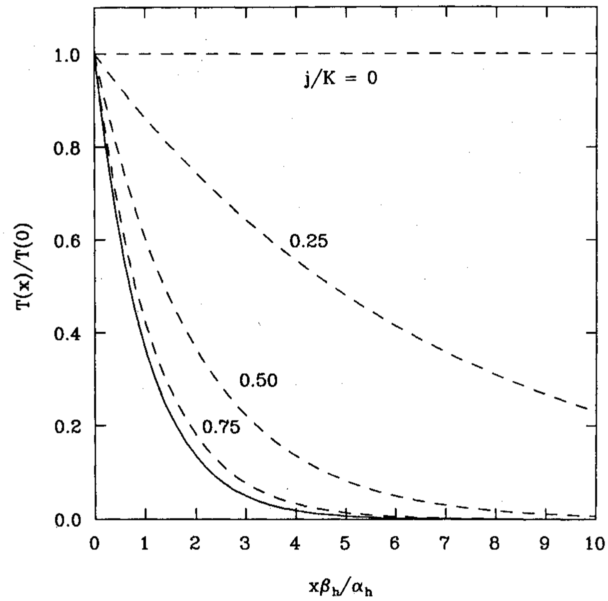

Fig. 2 Decay of steady and unsteady transverse temperature modes with distance: --- steady transverse modes; —— unsteady transverse modes.

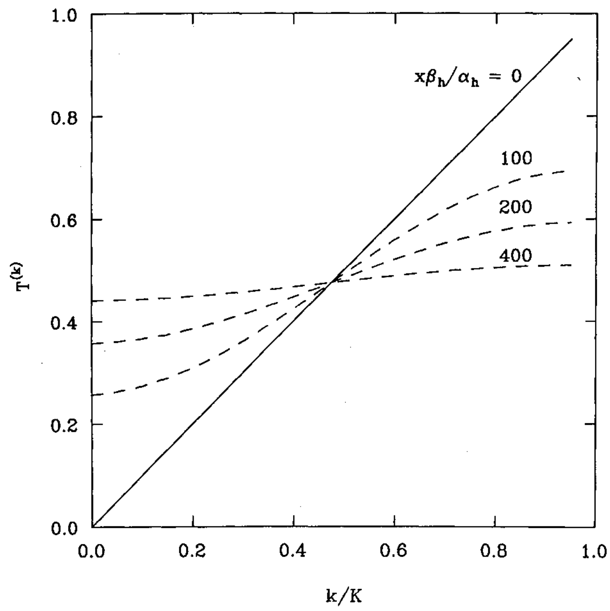

Fig. 3 Decay of linear transverse temperature gradient with distance.

for this behavior is that the linear gradient has a significant amount of low-frequency spatial content and therefore decays slowly according to Eq. (30).

One heat exchanger design strategy is to utilize a cross-parallel flow configuration without a separate thermalizer. The cross-parallel design has a lower thermal effectiveness than a cross-counterflow configuration with the same operating parameters; however, this is compensated for by the fact that the cross-parallel configuration has smaller exit temperature differences between gas and coolant, leading to smaller thermal wake amplitudes downstream of the heat exchanger. The data in Figure 2 suggest an alternative design approach: a short cross-counterflow heat exchanger with a relatively large exit temperature difference between gas and coolant in tandem with a thermalizer of the same fin density downstream of the heat exchanger. Because of the higher thermal effectiveness of the counterflow configuration, coolant pumping requirements are minimized. The thermalizer serves to efficiently smooth out the temperature variations in the fin wakes of the heat exchanger, which have the same spatial frequency as the thermalizer fins ($j/K = 1$) and thus decay rapidly in the thermalizer.

In closing, it should be noted that the theory developed here uses "macroscopic" heat exchangers theory, i.e., it is based on lumped heat transfer resistances and does not consider the details of the temperature and velocity distributions in individual passages; the same limitation is present in the analysis of Kulkarny et al.[8] It has been implicitly assumed that the heat is the same in both the steady and unsteady cases. In general this is not true, and it is important to understand the nature of the errors introduced by this assumption. The author has demonstrated in a related work[10] that the heat transfer coefficient in unsteady flow is bounded from below by the steady heat transfer coefficient. Thus the actual decay rates are larger than those predicted by the analysis presented here. In the context of laser heat exchanger design, this is a conservative error since the objective in this case is to minimize temperature variations.

Acknowledgments

The author is grateful to his colleagues W. J. Thayer, for introducing him to the problem, and V. C. H. Lo, for helpful discussions during the course of the work. In addition, the referees provided a number of useful suggestions. This work was supported by internal research and development funds from Spectra Technology Inc.

References

[1]Cassady, P. E., "Fluid Dynamics in Closed-Cycle Pulsed Lasers," *AIAA Journal*, Vol. 23, Dec. 1985, pp. 1922–1936.

[2]Kays, W. M., and London, A. L., *Compact Heat Exchangers*, 2nd ed., McGraw-Hill, New York, 1964, pp. 11–101.

[3]Cima, R. M., and London, A. L., "The Transient Response of a Two-Fluid Counterflow Heat Exchanger—The Gas-Turbine Regenerator," *Transactions of the American Society of Mechanical Engineers*, Vol. 80, 1985, pp. 1169–1179.

[4]London, A. L., Biancardi, F. R., and Mitchell, J. W., "The Transient Response of Gas-Turbine-Plant Heat Exchangers—Regenerators, Intercoolers, Precoolers and Ducting," *Transactions of the American Society of Mechanical Engineerings*, Vol. 81, 1959, p. 433.

[5]London, A. L., Sampsell, D. F., and McGowan, J. G., "The Transient Response of Gas Turbine Plant Heat Exchangers—Additional Solutions for Regenerators of the Periodic-Flow and Direct-Transfer Types," *Transactions of the American Society of Mechanical Engineers*, Vol. 86, 1964, p. 127.

[6]Rizika, J. W., "Thermal Lags in Flowing Incompressible Fluid Systems Containing Heat Capacitors," *Transactions of the American Society of Mechanical Engineers*, Vol. 78, 1956, p. 1407.

[7]Paynter, H. M., and Takahashi, Y., "A New Method of Evaluating the Dynamic Response of Counterflow and Parallel Flow Heat Exchangers," *Transactions of the American Society of Mechanical Engineers*, Vol. 78, 1956, p. 749.

[8]Kulkarny, V. A., Magiawala, K. R., and Schwartz, J., "Analysis of Thermal Homogenizers for Closed-Cycle Excimer Laser Flows," AIAA Paper 85-1602, July 1985.

[9]Hamming R. W., *Numerical Methods for Scientists and Engineers*, 1st ed., McGraw-Hill, New York, 1962, pp. 67–77.

[10]Cousins, A. K., "Bounds on Heat Transfer in a Periodic Graetz Problem," *Journal of Heat Transfer* (submitted for publication).